THERMAL CONDUCTION AND RADIANT ENERGY TRANSFER IN STATIONARY, HEATED AIR*

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Abstract—The relative importance of thermal conduction and radiation in stationary, heated air has been estimated for the diffusion and transparent gas approximations. Representative applications of the appropriate heat-transfer parameters are briefly considered.

NOMENCLATURE

- e, electronic charge;
- k, Boltzmann constant;
- $\bar{k}_{L, Pl}$, Planck mean absorption coefficient;
- $k_{L, Ro}$, Rosseland mean absorption coefficient:
- $I_{L, Pl}$, Planck mean free path;
- \bar{l}_{L, R_0} , Rosseland mean free path;
- L, characteristic length;
- *m*, electronic mass;
- \bar{m} , mean particle charge;
- n_i , concentration of species i;
- *N*, concentration;
- P_e , electron partial pressure;
- T, temperature;
- z, ion charge;
- \bar{z} , mean ion charge.

Greek symbols

- a_1, a_2 , similarity parameters measuring the ratio of the heat transfer by thermal conduction to heat transfer by radition;
- δ_T , a coefficient given in [10];

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- θ , temperature in electron volts;
- λ , coefficient of thermal conductivity;
- ν , frequency;
- ρ , density;
- σ, Stefan-Boltzmann constant.

Subscripts

- 0, standard or characteristic value;
- ν , spectral value.

I. INTRODUCTION

THE GREAT current interest in radiation gas dynamics and related topics is attested to by the density of publications in this field [1-4].

The similarity parameters in radiation gas dynamics are well known and have been discussed in several recently published papers [5, 6]. Among the less obvious results is the observation that the similarity groups for (nongrey) line radiation in non-isothermal systems are essentially the same as those for the corresponding isothermal radiators [6].

Rather than employing the dimensionless groups of radiation gas dynamics [5, 6], we may estimate directly the relative importance of conductive and radiative energy transport in heated air in order to gain some insight into the conditions under which radiative energy transport must be considered in stationary systems. This procedure is used in the following computations. It should be noted that the results are directly applicable to flow problems with radiant energy transport only in the Rosseland (diffusion) limit since, in this case, the radiative and conductive heat-transfer coefficients are additive. In the transparent gas limit, the important similarity group measuring the ratio of radiative energy loss from the system per unit surface area to the free stream rate of enthalpy transport per unit area is the more meaningful parameter in gas-dynamic studies and has been used in previously published papers, particularly by Goulard [5].

II. DEFINITION AND CALCULATION OF THE DIMENSIONLESS PARAMETERS

In the diffusion approximation, the ratio of heat transfer by thermal conduction to heat transfer by radiation is given by the relation

$$a_1 = \frac{3\lambda}{16\sigma T^3 \, \tilde{l}_{L, Ro}};\tag{1}$$

for transparent gases, this ratio is

$$a_2 \simeq \frac{(1/L) \lambda (\nabla T)}{4\sigma T^4 \bar{k}_{L, Pl}}.$$
 (2)

Here $\lambda = \text{coefficient}$ of thermal conductivity, $\sigma = \text{Stefan-Boltzmann constant}$, T = absolutetemperature, $\bar{l}_{L, Ro} = \text{Rosseland}$ mean free path, L = characteristic length, $\nabla T = \text{typical}$ temperature gradient, and $\bar{k}_{L, Pl} = \text{Planck mean}$ absorption coefficient.

Values of $\bar{l}_{L, Ro}$ and $\bar{k}_{L, Pl}$ have been obtained from the data listed in [7] where only the continuum contributions* to the spectral absorption coefficient of air were used. The coefficients of thermal conductivity for temperatures below 2 eV were taken from the work of Peng and Pindroh [8], who used a shielded Coulomb potential for electron-ion interactions. These estimates for λ are considered to be more accurate than those derived earlier by Hansen [9]. For temperatures of 2 eV and higher, the thermal conductivity of the electron gas is the dominant contribution to the total thermal conductivity. The coefficient of thermal conductivity was assumed to be given by the formula [10]

$$\lambda = \frac{80k^3 \left(2k/m\pi\right)^{1/2} T^{5/2} \delta_T}{\pi e^4 \,\bar{z} \ln \left[9k^4 \,T^4/4e^6 \,\pi \,\bar{z}^2 \,P_e(1+\bar{z})\right]}$$

where k = Boltzmann's constant, T = absolutetemperature, $\delta_T = a$ coefficient calculated numerically as a function of \bar{z} in [10],* $e = \text{elec$ $tronic charge}$, $\bar{z} = \text{mean ion charge} = \sum n_i z_i^2 / \sum n_i z_i$, $P_e = \text{electron partial pressure, and}$ $n_i = \text{concentration of particles with charge } z_i$. The expression for λ applies in the presence of d.c. current in a completely ionized gas. For partially ionized gases, corrections to the Spitzer and Härm [10] relation may be found in [11] but these can be shown to be unimportant for our purposes.

Numerically, the equation for λ becomes

$$\lambda = \frac{2.465 \times 10^{6} \,\theta^{5/2} \,\delta_{T}}{\bar{z} \log_{10} \left[\frac{2.401 \times 10^{20} \,\theta^{3}}{\bar{z}^{2} \,\bar{m} \,(1 + \bar{z}) \,N} \right]} \quad \frac{\text{erg}}{\text{cm-s degK}} \quad (3)$$

where θ = temperature in eV, δ_T has been defined previously, \overline{m} = mean particle charge = $\sum n_i z_i / \sum n_i$, and N = particle concentration = $5 \cdot 38 \times 10^{19} \rho / \rho_0$ (where ρ is the mass density and $\rho_0 = 1 \cdot 293 \times 10^{-3}$ g/cm³). If an electric field builds up sufficiently to restrain the flow of electric current, the values of λ from equation (3) should be multiplied by 0.4. Values of λ for air, as calculated (for $T \ge 2$ eV) from equation (3) and taken (for T < 2 eV) from [8], are shown in Fig. 1 as a function of temperature and density. In many practically important problems, equation (3) is known to yield values for the total thermal conductivity that are good to within about a factor of two.

In Table 1 are listed the values used for λ , $\bar{l}_{L, Ro}$, and $\bar{k}_{L, Pl}$ and the parameters α_1 and $\alpha_2 L/\nabla T$ calculated from equations (1) and (2). The dependence of α_1 on temperature and density

^{*} The method of Seaton [M. J. SEATON, Thermal inelastic collision processes, *Rev. Mod. Phys.* **30**, 979, 989 (1958)] which has been discussed also by Zhigulev, *et al.* [V. N. ZHIGULEV, YE. A. ROMISHEVSKII, and V. K. VERTUSHKIN, Role of radiation in modern gas dynamics, *Amer. Inst. Aeronaut. Astronaut. J.* **1**, 1473–1485 (1963) translated from *Inzh. Zh.* **1**, 60–83 (1961)] has been shown to give results that do not differ greatly from those calculated by using a simple, approximate procedure which we have described elsewhere (S. S. PENNER and M. THOMAS, Approximate theoretical calculation of continuum opacities, presented at the *AIAA* Aerospace Sciences Meeting in New York, January, 1964). At elevated temperatures and low densities, the line radiation

^{*} The coefficient δ_T is obtained by interpolation from Table III of [10].



FIG. 1. The coefficient of thermal conductivity for air at temperatures to 230 000°K for various densities. Data below 2 eV were taken from [8]; data above 2 eV were calculated from equation (3).

is shown in Fig. 2. The initial rise of a_1 with temperature is caused by the rapid decrease with temperature of $l_{L, Ro}$. However, as is shown in Fig. 12 of [7], $\bar{l}_{L, R0}$ increases sharply at temperatures above 10 eV at high densities and at somewhat lower temperatures for lower densities: hence a1 decreases above a well-defined temperature (see Fig. 2). The increase in $\bar{l}_{L, Ro}$ occurs as a result of the shift in the maximum of $k_{\nu, L}$ (the spectral linear absorption coefficient) toward higher frequencies (i.e. away from the maximum of the Rosseland weighting function). This shift results from bound-free contributions of nitrogen and oxygen ions. Thus the peak of the spectral absorption coefficient of nitrogen occurs at $h\nu/kT \simeq 6.0$ and 10.0 for kT = 5 and 10 eV, respectively, at a number density of 1017 cm⁻³. The maximum of the Rosseland weighting function occurs at $h\nu/kT = 3.83$.

The variation of $a_2L/\nabla T$ with temperature and density is shown in Fig. 3. The Planck mean absorption coefficient rises at low temperatures



FIG. 2. The coefficient a_1 for optically thick air as a function of temperature and density; θ is expressed in eV.

which, when combined with the $(1/T^4)$ factor in equation (2), causes a downward trend in α_2 . At high temperatures, $\bar{k}_{L, Pl}$ decreases sharply and, therefore, the parameter α_2 tends to increase.

III. REPRESENTATIVE APPLICATIONS OF THE HEAT-TRANSFER PARAMETERS TO PROBLEMS OF ENGINEERING INTEREST

The numerical values of $\tilde{l}_{L, R0}$ and $\tilde{l}_{L, Pl}$ $[\equiv (\tilde{k}_{L, Pl})^{-1}]$ in Table 1 determine the validity of the diffusion or transparent gas approximations. If L is a characteristic length of the system, the condition $\tilde{l}_{L, R0} \ll L$ implies that the diffusion approximation is applicable; similarly, the condition $\tilde{l}_{L, Pl} \gg L$ indicates the validity of the transparent gas approximation. It is of some interest to consider the relative importance of

θ (eV)	$\log_{10}\left(\rho/\rho_0\right)$	λ* (erg/cm-s degK)	l_L, R_0^{\dagger} (cm)	a ₁	$k_{L}, p_{l_{+}}^{+}$ (cm ⁻¹)	<i>Ī</i> L, <i>P</i> 1 (cm)	$a_2 L/\nabla T$ (cm²/degK)
	1	2-30+5	5-5+1	2.6-5	1.0	1.0	2-3-7
	ō	3.00+5	1.1+3	1.7-6	4.0-2	2.541	7.6-6
0.7	1	3.17+5	1.5+4	1.3-7	3.03	3.3+2	1.1-4
•	-2	1.609+5	2.0+5	5.0-9	1.7-4	5.9+3	9.6-4
	-3	1.076+5	3.0+6	2.2-10	8.0-6	1.2+5	1.4-2
	-4	1.281^{+5}	3.0+7	2.6-11	3.0-2	3.3+6	4.3 -1
<u></u>	1	3.25+5	7.0-1	9.8-4	2.0	5.0-1	4·0 ⁻⁸
	0	2.14^{+5}	1.4+1	3.2-2	1.0-1	1.0+1	5.2-7
1.0	1	1-923+5	$2 \cdot 0^{+2}$	2.0-6	7·0 ^{−3}	1.4+2	6.7-6
	-2	2.45+5	6·0+3	8.6-8	4.0-4	2·5+8	1.5-4
	-3	2.69^{+5}	8.0+4	7·1-9	$3 \cdot 5^{-5}$	2·9 +4	1.9-3
_	-4	2.20+5	1.3+6	3.6-10	1.4-6	7-1 +5	3.8-2
	1	2.50+6	2.1-2	3.1-2	$2 \cdot 2^{+2}$	4.5-3	1.7-10
	0	2.02+6	3.3-1	1.6-3	1.7+1	5.9^{-2}	1-8-9
2.0	1	1.335+6	1.0^{+1}	3-5-5	9.0-1	1.1	$2 \cdot 3^{-8}$
	-2	9·64 ⁺⁵	5.5^{+2}	4.6-7	$1 \cdot 0^{-2}$	1.0+2	1.5-6
	-3	7·37 ⁺⁵	2.5+4	7·8-*	2.3 4	4.3+3	4.9 -5
	-4	6.53+5	8·0 ⁺⁵	2.2-10	4.5-6	2.2+ 5	2.2.3
	1	2.56+7	2.7-3	1.6-1	3.5+2	2.9-3	2.8-11
	0	1.554+7	1.4-1	1.9-3	1.4+1	7.1-2	4.3-10
5.0	-1	1.045+7	6.2	2.8-5	4.0-1	2.5	1.0-8
	-2	7.37+6	$2 \cdot 8^{+2}$	4.5-7	9.0-3	1.1 **	3.2-
	-3	5.44+6	1.9+4	4.8-9	1.3-4	7.7+3	1.6-0
	4	4.31+6	2.0+8	3 6-11	3.0-6	3.3+5	5.6-4
	1	1.312+8	4·0 ⁻³	6.9-2	2.7^{+2}	3.7-3	1.2
	0	7.32+7	1.7-1	9-1-4	1.0+1	1.0-1	1.8^{-10}
10.0	1	4.66+7	9.0	$1 \cdot 1^{-5}$	$2 \cdot 0^{-1}$	5.0	5.7-9
	-2	3.20+7	5.0^{+2}	1.4-7	4 ·0 ^{−3}	2.5^{+2}	1.9-7
	3	2.36+7	4·0 ⁺⁴	1.2-9	7·0 ^{−5}	1.4+4	8.2-6
	4	1.860+7	3.8+6	1.0-11	9.0-7	1.1+6	5-0 -4
	1	3.93 +8	7.0-3	3.5-2	1.8+2	5.6-3	1-0 11
	0	1.702+8	3.0-1	3.6-4	5.0	2.0-1	1.6 10
15.0	1	1.056+8	$2 \cdot 1^{+1}$	3.2-6	8.0^{-2}	$1 \cdot 2^{+1}$	6.3 \$
	2	7.41+7	2·1+3	$2 \cdot 2^{-8}$	1.0 -3	1-0+3	3.6 '
	-3	5-67+7	1.9+5	1.9^{-10}	1.0-5	1.015	2.7 *
. <u> </u>	-4	4.59+7	1.0+7	2.9-12	1.2-7	8.3 **	1.8
	1	5-40+8	1.7-2	8.4-3	1.1+2	9.1-3	7.5^{-12}
	0	2.92+8	8.0-1	9.7.5	2.0	5.0-1	2.2-10
20.0	-1	1.878+8	6·0 ⁺¹	8.3-7	2.3-2	4.3+1	1.2.**
	2	1.365+8	5·0+8	7.2-8	3.0-4	3-3+3	6.9.1
	-3	1.071+8	5.0+5	5.7-11	3.0-	3.3 **	⊃·4 […] "
	4	8.76+7	$2 \cdot 1^{+7}$	1.1-12	3.0-0	3.3*1	4.4

Table 1. Heat-transfer parameters for high-temperature air

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Note: $\rho = \text{air density}, \ \rho_0 = 1.293 \times 10^{-3} \text{ g/cm}^3$. Superscripts denote multiplication by the corresponding power of ten.

* For T = 0.7 and 1.0 eV, λ was found from the data of [8]; for T > 1.0 eV, λ was calculated from equation (3). † Values of \tilde{l}_{L, R_0} were obtained from Fig. 12 of [7].

‡ Values of \vec{k}_L , p_l were obtained from Fig. 11 of [7].



FIG. 3. The coefficient $\alpha_2 L / \nabla T$ for optically thin air as a function of temperature and density; θ is expressed in eV,

radiant energy transfer in various regimes of gas dynamics.

 $L \simeq 0.01$ cm. This value for L is of the order of magnitude of the wall boundary-layer thickness in a shock tube from 1 to 100 µs after the passage of the shock front; alternatively, L applies to the thermal layer thickness formed on a fast (velocity $\simeq 35\,000-40\,000\,$ ft/s, $\rho/\rho_0 = 1$ to 10) atmospheric entry vehicle. For density ratios of 1 to 10 and temperatures below 2 eV, $\tilde{l}_{L,R0}$ varies from 0.02 to 1.1×10^3 cm and $l_{L, Pl}$ varies from 4.5×10^{-3} to 25 cm. The diffusion approximation is therefore not valid whereas the transparent gas approximation applies only at the lower temperatures. Assuming a temperature drop of 10000°K across the thermal layer, Fig. 3 shows that a2 varies from 40 to 500 (corresponding to T = 1 eV, $\rho/\rho_0 = 10$

or 1), i.e. radiation energy transfer is relatively unimportant. Under more extreme conditions $(\rho/\rho_0 = 10, T = 2 \text{ eV}), a_2 \simeq 0.02$, but the transparent gas approximation now begins to become invalid since $\bar{l}_{L, Pl} \simeq 4.5 \times 10^{-3}$ cm.

 $L \simeq 0.1-1.0$ cm. This estimate for L is of the order of magnitude of the thermal layer thickness on an atmospheric entry vehicle at high altitudes (altitude $\simeq 100\ 000$ ft, velocity $\simeq 35\ 000$ ft/s, $\rho/\rho_0 \simeq 10^{-1}$). The gas is transparent, $a_2 \gg 1$, and radiation heating is relatively unimportant.

 $L \simeq 1.0-1000$ cm. This range of values for L corresponds to typical vehicle dimensions. For $T \simeq 0.7$ eV, $\rho/\rho_0 = 0.1$, L = 10 cm, $\nabla T = 10^3$ degK/cm, it is found that $\alpha_2 = 1.1 \times 10^{-2}$ and the transparent gas approximation is valid. In this case, radiation losses are more important than conduction losses.

 $L \simeq 10^{3}-10^{4}$ cm. These lengths correspond to characteristic diameters of initial fireballs of nuclear bombs. Taking $T \simeq 10$ eV, $\rho/\rho_{0} = 0.1$ and $L = 10^{4}$ cm, we find from Table 1 that $\tilde{l}_{L, Ro} = 9$ cm and, therefore, the gas is optically thick. Also $\alpha_{1} = 1.1 \times 10^{-5}$ and, for this reason, conductive heating is completely negligible inside the fireball.

 $L > 10^4$ cm. The next range of interest is mainly of astrophysical importance (we use air estimates since the continuum radiation in heated plasmas is not very sensitive to chemical composition). The values of $\tilde{l}_{L, Ro}$ for air as found in Table 1 for low densities and T = 20 eV will give an idea of some phenomena that may be important. We find $\tilde{l}_{L, Ro} \approx 100$ miles and $a_1 \leq 1$ so that, as is well known, the diffusion approximation is valid for the atmospheres of stars and conductive energy transport is of lesser importance.

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Résumé—L'importance relative de la conduction thermique et du rayonnement dans de l'air chauffé et immobile a été estimée pour les approximations de la diffusion et du gaz transparent. Des applications typiques des paramètres convenables de transport de chaleur sont considérées brièvement.

Zusammenfassung—Der relative Einfluss der thermischen Leitung und Strahlung in ruhender, beheizter Luft wirde für die Näherung diffusen und transparenten Gases abgeschätzt. Besondere Anwendungen der geeignetsten Wärtmeübergangsparameter sind kurz erörtert.

Аннотация—Дается оценка относительной доли теплопроводности и радиации тепла в неподвижном нагретом воздухе для определения диффузии и прозрачного газа. Кратко рассмотрена применимость соответствующих параметров переноса тепла.